# Techniques for Algorithmic Composition

A brief Treatise outlining processes for manipulating melodic, harmonic and rhythmic progressions

By David Matthew Shere

# TECHNIQUES for ALGORITHMIC COMPOSITION

by David M. Shere; copyright © 2004

**TABLE OF CONTENTS**

## I. Introduction

All composition is derivative in some sense. I would not dream of making any statements to the contrary, or trying to convince anyone otherwise; to do so would be patently dishonest. Historically, it is impossible to say exactly when and where music began, so it is equally impossible to credit any human source with its "invention". Music exists in nature independently of man; hence, it was certainly "discovered"- rather than "invented"- by our prehistoric ancestors attempting to mimic the whistlings of birdsong, or imitate the rhythmic patterns of some galloping equine species. Furthermore, as composers, we are usually inspired in our initial efforts to write music by hearing the music of others, so our first creative attempts are nearly always blatantly imitative. My own earliest musical scribblings sound very much like third-rate impersonations of Bach or Beethoven. The point is that it is essential to give credit where credit is due.

Having said all that, I do believe that an experienced artist may eventually reach a stage of technical development where it is possible to have creative insights of a unique and independent nature. If that artist has "done their homework", studied what is known of existing techniques, and worked diligently to assimilate the state of the art and apply it in their own creative output, there may come a point where new discoveries are demonstrated concerning the theoretical nature of the medium. In music, the best examples are of course the most obvious: J.S. Bach's *The Art of Fugue*, Alexandre Scriabin's Impressionistic works based on his "mystic chord", Arnold Schoenberg's codifications of 12-tone techniques, Olivier Messiaen's modes of limited transposition and additive rhythmic cycles, Nicholas Slonimsky's *Thesaurus of Scales and Melodic Patterns*, Gyorgy Ligeti's "micropolyphony", John Cage's use of chance operations and the *I Ching*, and so forth. Each of these musical milestones has in common the demonstration of theoretical principles that are firmly grounded in historical precedent, and at the same time present new and unique discoveries of the nature of how music works on a theoretical level, with inspiring results in creative application at the hands of the particular discoverer-composer.

In discussing theoretical evolution in the musical medium, notice that I am emphasizing the term "discovery" as opposed to the term "invention". This is not to minimize the accomplishments of the composers in question, but simply to clarify a point: I firmly believe that all of music, as we know it, is pre-existing. The sounds that we use, as composers, to paint our tone-pictures have been present in nature since the dawn of time. How we *interpret* those pre-existing elements into phrases of melody and harmony and formal structure is what defines us as individual composers, but the elements were there to be found. Anything that each of us feels is unique to our own work or compositional language could just as easily have been discovered by some other practitioner.

I feel very strongly that it is *essential* to recognize this distinction about the way we composers interact with our medium, in order to maintain a sense of humility and perspective about our work, and to view the tools we use and the results of our labors with an appropriate sense of proportion. Within the past several decades, there has been far too much self-indulgent nonsense expressed by far too many mediocre composers (and mediocre artists in other disciplines as well) concerning the self-important nature of their "creative" output. "My work is Great because I say so," asserts the amateur (and immature) composer/artist, in defense of indolent thinking, lack of discipline and a non-existent work ethic. I would dare to flatly negate this idea. The "greatness" of a work of art is not a characteristic that can be determined by its creator, for to even attempt to do so places the artist in the untenable position of self-fulfilling narcissism, a position which has consistently failed many mediocre artists throughout recorded history. Ultimately, the issue of whether or not a work of art exhibits "greatness" can only be addressed by someone other than the artist, via the following questions: "Is this artwork worth preserving beyond the lifetime of its creator? Is it worth learning from and building upon? If so, why?"

Therefore, it is my fundamental belief that the focus of an artist should not be on the attainment of a status of "greatness", but rather on the processes of creation and discovery. I truly believe in seeking fulfillment through creating art for Art's sake, and *not* through any adulation or financial success that might be attained as a by-product of one's creative efforts. The focus of a true artist should be on the *process of discovery itself*, and not on any happenstance notoriety, fortune, or status which may result therefrom. One should not spend one's life asking the question, "How may I achieve greatness?" I ask you to consider this question instead: **What have you discovered?**

In this treatise, I hope to share with the reader some technical discoveries I have made in the course of my own compositional meanderings. These are not novel ideas. Each of the following concepts can be shown to have firmly rooted ties to the pre-existing work of other composers; I wish to make that absolutely clear from the outset. By no means would I make the claim that any of these concepts are unprecedented, or have been developed in a creative vacuum, so to speak. My purpose in outlining these thoughts is not to call attention to myself, but rather to call attention to a number of algorithmic processes that I make use of, which I believe are potentially unique and logical manifestations of the evolutionary nature of music composition. The notability of each idea is for the most part extremely subtle in nature: these are processes that, while familiar, are just distinct enough from their precursors to merit further exploration and study, in my humble opinion. Only the passage of time, and the perspective of objective observers, will determine whether my opinion is in fact correct.

I owe the development of each of these ideas not only to the work of those composers who inspired them, but also to the patience and diligence of all the teachers who have pointed me in the right direction, and challenged my evolution at each step. **In particular, I wish to thank Dr. Curtis Roads**, whose mastery of algorithmic compositional processes and willingness to act as a sounding-board for my ideas provided the necessary space for me to hash out the grammar and semantics of each technical concept; **and Dr. Jeremy Haldyna**, who coined the term "contour cycling", and whose groundbreaking compositional work involving mathematical systems based on Mayan artifacts is both epic and monumentally inspiring.

-David M.Shere

September 2, 2004

## II. A Brief Explanation of the Format

As this is the first draft of this treatise, each key concept will be explained as concisely as possible, and briefly illustrated with accompanying figures and/or musical examples from my own work. The verbal explanations comprise the main body of the paper; the musical examples can be found in the first appendix. The second appendix contains a template of a checklist of chromatic aggregates; the third appendix contains the bibliography.

**With regard to the ordering of topics:** Each concept has been presented more or less in order of historical precedent, evolution and degree of complexity, beginning with topics that apply to neo-tonality and centricity, and progressing to more complex dodecaphonic procedures. I have also made every effort to distinguish, by order of presentation and sub-grouping, those topics which apply to manipulating **harmony** from those topics which apply to manipulating **melody**; however, there is a certain amount of overlap between the two ideas, and it is not possible to *completely* separate these topics by this distinction. Processes which apply to manipulating harmony may also be applied to manipulation of melody, and vice versa.

## III. Algorithms for Manipulating Harmony

### 1. Expanded Functional Chord Progressions

Of all the elements which make up music, unquestionably one of the most difficult elements to produce is a well-constructed *harmonic progression.* Chords in and of themselves hold a certain amount of interest, particularly chords containing alterations and extensions [**example 1-1**]. However, the interest of any chord in the ear of the listener- even a simple triad- can increase exponentially in the context of a well-thought out series of harmonic relationships, and therefore the study of chord progressions is of paramount importance. For the purposes of illustration, let us assume the following progression:

[I-iii$^{7}$-vi$^{7}$-IV$^{7}$-ii$^{\varnothing 7}$-V$^{7}$-i] **[ex. 1-2]**.

This progression is not particularly interesting; however, it is necessary as a foundation for the following operations. Having established this chord progression as a basic harmonic framework, each chord within the progression can then be treated as a new harmonic center, or *point of resolution.* One may then proceed to elaborate upon each point of resolution by interpolation, applying a secondary chord progression to each established chord, thus:

{[vi$^{7}$-ii$^{7}$-V$^{7(\flat 9)}$-**I**][vi/iii-ii$^{\varnothing 7}$/iii-V/iii-**iii**$^{7}$][ii$^{o7}$/vi-V$^{65}$/V/vi -V/vi-**vi**$^{7}$][N$^{6}$/IV-IV$^{4(\flat 3)}$-V/IV-**IV**$^{7}$][Ger.6/ii-V/ii-$\flat$ii$^{o7}$/ii-**ii**$^{\varnothing 7}$][N$^{6}$/V-V$^{43}$-V/V-**V**$^{7}$][Fr.6-I$^{64}$-vi-IV-I$^{64}$-V-**i**]} **[ex. 1-3]**

This progression is, again, not especially interesting or unusual, but it effectively demonstrates the mechanism and intent of the process. Rather than merely using *applied chords* to embellish our progression, we are using entire "*applied progressions*" to embellish each chord, creating "progressions within progressions".

Once the new extended progression is established, one may yet again treat every chord within the new progression as another point of resolution, creating applied progressions for each new chord center [**ex. 1-4**]. I would venture to suggest that, in this third stage of the operation, we have finally arrived at results that are genuinely interesting and unusual. It can be considered obvious at this point in the exercise that the process of removing the applied progressions further and further from the initial tonicism lends itself well to creating extremely chromatic and dissonant passing chords and basslines, with little or no reference to diatonic functionality other than voice-leading [see sections 2 and 3 for elaboration]. The operation can be repeated as many times as the practitioner wishes, creating as many hierarchies of harmonic relativity as one chooses. Creating applied progressions constitutes a *potentially infinite recursive algorithm.*

Furthermore, one may manipulate every individual chord in the final result by voicing and bass-note inversion, by adding extensions and/or alterations, and/or by changing the nature of the basic triads wherever applicable or desired. The entire progression can be manipulated via transposition, retrograde [R], inversion [I], or retrograde inversion [RI]. The results can become so ambiguous that they completely mask their functional origin, without destroying the underlying integrity of the elementary relationships in the slightest. In effect, this technique can be viewed as the endless chromatic embellishment of a functional diatonic chord progression.

**In practical creative application:** once all chosen operations are finished, the completed progression may be used as a *template* with which to guide the formal harmonic structure of a composition. The entire progression may be used verbatim, or sections that are particularly appealing (including the bassline alone, the melody alone, an inner voice, or any combination of voices) may be extracted and used in isolation. Segments can be cycled in repetition, or sequenced by transposition. Each chord/harmonic center may be used as a basis for creating figurations and melody. The overall harmonic rhythm of the form can be manipulated as desired, creating areas of prolonged static harmony or rapid cycles of resolution from one chord to the next.

Examples of this compositional approach can be found in the works of Beethoven, Bach, Ravel, Brahms, Chopin and countless others. However, in this monograph the degree of infinite chromatic variability and potential dissonance that can be achieved via this method can be clearly seen in a modern context.

**1. SUMMARY:**

- Write a chord progression.
- Treat each chord in the progression as a new harmonic center [or *point of resolution*].
- Create applied progressions for each chord, to be interpolated into the existing progression.
- Write out the full results.
- Repeat the entire process for each new completed progression, as many times as necessary.

**2. Non-functional diatonic harmonies**

An interesting and useful by-product of the previous operation is the potential generation, via passing chords, of diatonic harmonies that are not necessarily being approached or resolved in a functional manner. Ordinarily, this type of harmony is referred to in the theoretical community as *"chromatic harmony"*; however, as I have found it necessary to reserve the term *"chromatic"* to refer to harmonies based on pitch-sets which have no diatonic or triadic origin, I will refer to these chord progressions as *"non-functional diatonic harmony"*. This phenomenon can be isolated, and utilized as the basis of a creative operation in and of itself. The object of this process is to devise a progression of diatonic chords that avoid traditional functional relationships, while still observing traditional voice-leading principles. Essentially, we are retaining many of the parameters of diatonic harmony while discarding traditional harmonic functionality.

The best method for approaching this problem is to once again map out a harmonic progression via roman numerals, **taking great care to avoid typical relationships** such as [ii-V], [iii-vi], [V/iv-iv], [I-IV-N$^6$-V], etc. (see **table 2-1** and **2-2** for further clarification and assistance). Basically, diatonic chords are being randomly selected and strung together, guided only by the following stipulation:

- **In a non-functional progression, a diatonic chord which has any direct, functional relationship to the previous diatonic chord *may not follow that chord.***

The following example is an illustration of this concept:

$[\mathrm{I}^{(add9)}\text{-}\mathrm{v}^{7}/\mathrm{iii}\text{-}\flat\mathrm{III}\text{-}\mathrm{V}^{7(\flat 9)}/\mathrm{ii}\text{-}\mathrm{v}^{(ma7)}\text{-}\mathrm{Fr.6}/\flat\mathrm{II}\text{-}\mathrm{vi}^{7}/\flat\mathrm{II}\text{-}\mathrm{V}^{7(\flat 9)}/\mathrm{vi}\text{-}\flat\mathrm{VI}^{7}\text{-}\mathrm{ii}^{7(\flat 9)}\text{-}\ \mathrm{ii}^{\varnothing 7}/\mathrm{V}/\mathrm{iii}\text{-}\flat\mathrm{iii}^{7}\text{-}\mathrm{Fr.6}](\mathrm{etc.})$

**[ex. 2-1]**

This operation has the wonderful result of generating great dissonances using diatonic chords which individually may not be perceived as particularly dissonant, but sound dissonant in this context due to their dramatic non-functional contrast. By juxtaposing diatonic chords at close proximity in such a paradoxical disorder of unresolution, one uses the subconscious expectations of the ear to great effect, simultaneously flirting with and abruptly defying those expectations. As a final note, the results of this process can be altered or manipulated via all of the same devices that were reviewed in section 1, *including* applying expanded progressions to each chord to extend the results.

**2. SUMMARY:**

- Chart out a diatonic chord progression using roman numerals, taking great care to avoid any semblance of traditional approach and resolution practices (see following tables for reference).
- Realize this chord progression, adhering as strictly as possible to rules of traditional voice leading (avoid parallel 5th's/8va's; strive for contrary motion between soprano and bass, etc.)

## TABLE 2-1: ROOT RELATIONSHIPS of BASIC TRIADS/DOM. 7ths to TONIC

[The root of any triad-based chord can be analyzed via roman numerals relative to ***any*** key center (whether it it is approached/resolved functionally or not), however obscure the connection might be. Some of these chords can be analyzed in several ways; I have chosen the most expedient analyses to show the simplest possible relationship of each chord root to the tonic. **To assemble a non-functional progression from the following chart, place chords next to each other which have no direct roman-numeral relationship.** Dominant 7ths which are listed as Ger.6 chords also function as secondary/applied dominant chords; be sure to avoid both functions.]

| **ROOT** | **#1/b2** | **2** | **#2/b3** | **3** | **4** | **#4/b5** | **5** | **#5/b6** | **6** | **#6/b7** | **7** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **Major/aug.** | $N^6$ | V/V | bIII | V/vi | IV | IV/$N^6$ | V | bVI | V/ii | bVII | V/iii |
| **minor** | iv/bVI | ii | ii/$N^6$ | iii | iv | iv/$N^6$ | v | vi/V/iii | vi | vi/$N^6$ | vi/V/V |
| **dim.** | vii°/ii | ii° | vii°/iii | vii°/IV | ii°/bIII | vii°/V | vii°/bVI | vii°/vi | ii°/V | ii°/bVI | vii° |
| **Dom. 7th** | Ger.6/iv | $V^7$/V | Ger.6/v | $V^7$/vi | $V^7$/bVII | Ger.6/bVII | $V^7$ | Ger.6 | $V^7$/ii | $V^7$/bIII | $V^7$/iii |

## TABLE 2-2: 7th CHORDs

[**7th chords,** and their various permutations, provide a great deal of additional harmonic obscurity in assembling a non-functional chord progression. **When using altered 5th's, keep in mind that:**

**1) only major triads can be altered;**

**2) (b5=#11) and (#5=b13), so only one or the other should be used in the same chord.**

The root functions of 7th chords (other than dominant 7th's, already listed above) are generally as follows:

- M7 functions as a major triad;
- m7 and $m^{(\triangle 7)}$ function as minor triads;
- $^{\varnothing 7}$ and $^{o7}$ function as diminished triads

In addition, keep in mind that **extensions and suspensions** can also be applied to pure triads without 7ths, and scale degrees {(9=2), (11=4), and (13=6)}, dependent on voicing and the presence or absence of the 3rd or 7th of a chord.]

| **Seventh Chords** | **Alterations** | **Suspensions/Extensions** |
|---|---|---|
| Dominant 7th | b5, #5 | b9, 9, #9, 11, #11, b13, 13 |
| Major 7th [M7, maj. 7, $^{\triangle 7}$] | b5, #5 | b9, 9, #9, 11, #11, b13, 13 |
| minor 7th [m7,min.7, $^{-7}$] | **n/a** | b9, 9, 11, #11, b13, 13 |
| minor/Major 7th [$m^{(\triangle 7)}$] | **n/a** | b9, 9, 11, #11, b13, 13 |
| half-diminished 7th [$^{\varnothing 7}$] | **n/a** | b9, 9, 11, b13, 13 |
| diminished 7th [$^{o7}$] | **n/a** | b9, 9, 11, b13, 13 |

**3. Micro-resolutions (or "tonal warp" harmony)**

Another interesting and useful by-product of expanded functional chord progressions is the potential generation of completely chromatic, non-diatonic passing chords, via the use of *pseudo-polyphonic* voice-leading. These are chords that may have some semblance of- or obscure basis in- tonality, but they cannot be analyzed from any traditional diatonic perspective due to: 1) a fundamental lack of an identifiable diatonic root; and 2) a fundamental lack of an identifiable triadic interval structure. They are *non-diatonic pitch-sets*, generated solely through chromatic voice-leading from one diatonic harmony to the next.

This phenomenon, like the previous one, can also be isolated, and utilized as the basis of an operation in and of itself. I have coined the term *"micro-resolutions"* to name this operation, in honor of the fact that this approach evolved largely due to the influence of Gyorgy Ligeti's *"micropolyphony"* (a compositional technique that involves generating dense polyphonic sound masses, using many semitone-based melodies each of which lies in a very small compass range). *"Tonal warp"* is the term I used to describe my less-sophisticated attempts at this harmonic approach, prior to discovering and studying the music of Ligeti.

Let us begin once again with the initial functional chord progression used in section 1 **[ex. 3-1]**. We will then interpolate between each chord a chromatic passing chord, in which every pitch moves from the previous chord to the following chord by orderly stepwise motion, and yet exhibits no discernible diatonic characteristics or structure whatsoever **[ex. 3-2]**. These are our *"chords of micro-resolution"*. Then, as with the first operation, we will develop this progression yet again by applying via interpolation another hierarchy of chromatic passing chords to each new point of resolution **[ex. 3-3]**. As a final means of achieving complete atonality, we will go back and alter the few initial diatonic chords chromatically, so that there is no longer any semblance of tonality left in the example **[ex.. 3-4]**. In this manner, we will have achieved a thoroughly dissonant, atonal chord progression that retains the illusion of functionality, because its underlying logic is still grounded in diatonicism no matter how far removed the results have become from the original progression.

**3. SUMMARY:**

- Create a functional or non-functional diatonic chord progression, paying careful attention to proper voice-leading.
- Create a completely chromatic, non-diatonic passing chord between each point of resolution, via stepwise voice-leading.
- Repeat the process as many times as necessary.

**4. Generating Random Diatonic Harmonies via Twelve-tone rows**

*12-tone rows* are generally made use of as self-contained units from which both chords and figuration are generated, cycling through the interval content of the row and its various permutations to construct harmonies and melodies. However, it is also possible to incorporate 12-tone rows into the structural fabric of *diatonic* music, without compromising the fundamental integrity of either the diatonic or the *dodecaphonic* aesthetic. The simplest way to go about this incorporation is to create a 12-tone row **[ex. 4-1]** and use it compositionally as either a *bassline* or a *soprano melody*, which can then be harmonized freely **[ex.'s 4-2, 4-3]**. It is worth noting at this point that, in a modern context, any given pitch can be harmonized with any given pitch-set, so harmonic choices are truly arbitrary at this stage.

In order to persuasively blend the aesthetics of both diatonic and dodecaphonic music while retaining the characteristic features of each, it is imperative that both one's choice in 12-tone material and one's subsequent choice in harmony should have common elements. Each should be reflective of the other to some degree. Thus, the 12-tone row should not be overly atonal in nature, and the harmonizations should not be overly simplistic or triadic. There should be subtle elements of diatonicism in the row, and subtle elements of chromatic ambiguity in the harmony. The goal is to create a musical result which is neither absolutely diatonic nor completely atonal, but a convincing marriage of both aesthetics.

**4. SUMMARY:**

- Create a 12-tone row with diatonic implications.
- Determine if it is to be used as a soprano melody or a bassline.
- Harmonize the melody or bassline freely, using diatonic or *pandiatonic* harmonies with strong chromatic inflections/embellishments.

**5. Generating Bitonal chord streams**

A variation of the technique described in section 2 can be used to generate *bitonal* chord progressions. Instead of mapping out a single horizontal stream of non-functional roman numerals for governing a chord sequence, this approach involves mapping out two concurrent streams of non-functional roman numerals, stacked vertically against each other, with the following added dimension:

- **The upper harmony should have no direct functional diatonic relationship to the lower harmony.**

In other words, a [V/ii] chord should not be placed over a [ii] chord; a [IV] chord should not be placed over a [I] or [vi] chord, etc. The rules of non-functional diatonic harmony that were discussed in section 2 now also apply to concurrent *verticalities.* In addition, the practitioner should seek to avoid or eliminate doubled *pitch-classes* in any of the verticalities, which may require some experimentation and revision to achieve. (Of course, these rules are somewhat arbitrary, and are in keeping with the aesthetic described thus far. There are other definitions of, and approaches to creating, bitonality).

The following chord chart illustrates a possible realization of this technique **[ex. 5-1]**:

**TABLE 5-1: Example of Bitonal chord streams**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $ii^{ø7}$ | $bVI^{9}$ | $V^{9}$/iii | $iv^{(\triangle 7)}$ | $bVII^{7}$ | $ii^{ø7}$/V | $bII^{7}$ | Fr.6/V | $ii^{ø7}$/IV | $V^{7}$/vi |
| $V^{7}$/iii | $IV^{7}$ | Fr.6 | $V^{(\triangle 7)}$ | iii add9 | $vi^{7}$/bII | $V^{7}$/ii | $ii^{ø7}$/ii | $vii^{o}$/iii | Fr.6/ii |

**6. Dodecaphonic Chord Progressions (4th/TT/5th progressions)**

Using symmetrical harmonies based on **interval class 5** (4th's/5th's) **[ex. 6-1]** and **interval class 6** (tritones: #4th's/b5th's) **[ex. 6-2]**, it is possible to create logical 12-tone chord progressions which can be used to harmonize almost any melodic construct, including diatonic melodies. This concept is the rational counterpart to section 4: rather than harmonizing dodecaphonic melodies using diatonic chords, we are harmonizing diatonic melodies using dodecaphonic chord progressions.

(**Before proceeding with this concept:** it is important to take note, at this point in the monograph, that working with any 12-tone process which uses more than 2 or 3 rows simultaneously requires an organized, systematic approach. The most useful tool that I have found for this purpose is to keep a checklist of chromatic scales close at hand **[appendix 2]**. This permits efficiency and rapidity in row construction, 12-tone analysis, and proof-reading for unwanted or accidental repetitions of pitch-class.)

To illustrate this operation, let us consider a simple diatonic major scale in the key of C. By drawing from our pre-constructed 12-tone chord sets and manipulating their contents accordingly, it is possible to harmonize the entire scale in stepwise fashion. When we place two transpositions of our basic chord sets next to each other **[ex. 6-3]**, it is immediately evident that we have a diatonic tetrachord in the upper voice. If we make two simple voice exchanges **[ex. 6-4, 6-5]**, we now have a logical dodecaphonic chord progression harmonizing the tetrachord [0,2,4,5], and containing two chromatic aggregates with no doubled or missing pitches. By duplicating this progression, transposing it at the 5th, and then adding it to the previous progression **[ex. 6-6]**, we have created a logical dodecaphonic chord progression which harmonizes a **C major scale**.

Of course, this type of harmonization can be accomplished using much simpler chord textures **[ex. 6-7]**. The advantage of the more complex approach is that it affords a density of harmonic texture that is nearly impossible to achieve any other way.

There are an unlimited number of ways to make use of these types of 12-tone chords. This particular operation simply demonstrates yet another direct, effective, practical method by which the gap between tonality and atonality may be bridged. The beauty of this approach is that the computational integrity of neither the dodecaphonic nor the diatonic aesthetic is compromised.

**7. Dodecaphonic Polytonality**

At this time, the most complex harmonic system which I have yet managed to contrive is one which I would term *"dodecaphonic polytonality"*. It combines, in part, a variation on the bitonal operation described in section 5, and the dodecaphonic chord constructs described in section 6. This approach has been evolving throughout my compositional efforts of the past five years or so, and came to full fruition in two recent works: "A Bowl of Greek Fire (sonata for electric guitar, piano and vibraphone/ percussion)", and "Florescence (piano quintet #2)".

As the development of this idea has been predominantly evolutionary, it has not yet reached a stage in its conception where I am capable of presenting a detailed description of the necessary technique for creating it. Therefore, we shall examine this idea by way of analysis rather than methodology. The following illustration is from movement I of the quintet, showing a figuration for the piano **[ex. 7-1]**. Notice that 3 separate harmonic constructs are implied in this passage: the arpeggiation in the bass clef, and two distinct broken chord constructs in the treble clef, the first in ascending motion and the second in descending motion. If we verticalize these harmonic entities **[ex. 7-2]**, it becomes immediately evident that each of the 3 harmonies occupies its own space, creates its own texture, and relates only to itself. Polytonality is achieved by unifying the rhythmic activities of each harmonic entity, simultaneously displacing them slightly via cross-rhythms to maintain their individual sonic space.Finally, as the entire chromatic aggregate is represented by the collective pitch content of these 3 distinct systems, a 12-tone harmonic construct emerges, justifying the term "dodecaphonic polytonality".

## IV. Algorithms for Manipulating Melody

### 8. Recursive Scalar Variations

Any ordered pitch set can be used as a source for generating melodic figuration. There are all sorts of traditional approaches for constructing melodic material: scalar/modal sequencing, motivic fragmentation and transposition, rhythmic variation, augmentation, diminution, through-composition, inversion, retrogradation, arpeggiation, pandiatonicism, dodecaphonicism, set theory, and so on. For the most part, the studies I will be presenting in these monographs deal with processes that are less concerned with pure melody (or *tunes*) than with *textural figuration*. In this first monograph, we will be examining the most fundamental operation that I use to produce contour-based *melodic gestures*, for which I have coined the term ***"recursive scalar variations"***.

*Recursive scalar variations* are an extremely simplistic type of melodic figuration of a specific nature that is not necessarily sequential in the traditional sense, and which gives the illusion of a potentially infinite, cyclical melodic line. As I see it, there are two types of **recursive components** (which are essentially just motivic scale or pitch-set fragments): 1) *half* or *partial* **elements of recursion [ex. 8-1]** (equivalent to the *prime* form of a melodic motive), and 2) *full* or *complete* **elements of recursion [ex. 8-2]** (equivalent to the combined prime and *retrograde* forms of a melodic motive, with the last pitch of the prime mapped onto the first pitch of the retrograde). By combining partial and complete elements of recursion, we can construct a single ***recursive cell*** **[ex. 8-3]** which, when duplicated and transposed correctly, can be dovetailed into itself, creating a continuous unbroken melodic pattern **[ex. 8-4]**. The results of this operation can be manipulated in any conceivable manner, from applying registral transposition to selected notes, to superimposing various rhythmic patterns, to deriving more traditional types of step-wise transposable sequences from the raw material [see section 9: "Sequence Filtering"]. Finally, a melodic gesture can be constructed using several recursive cells in succession, to avoid repetition of the motivic contour.

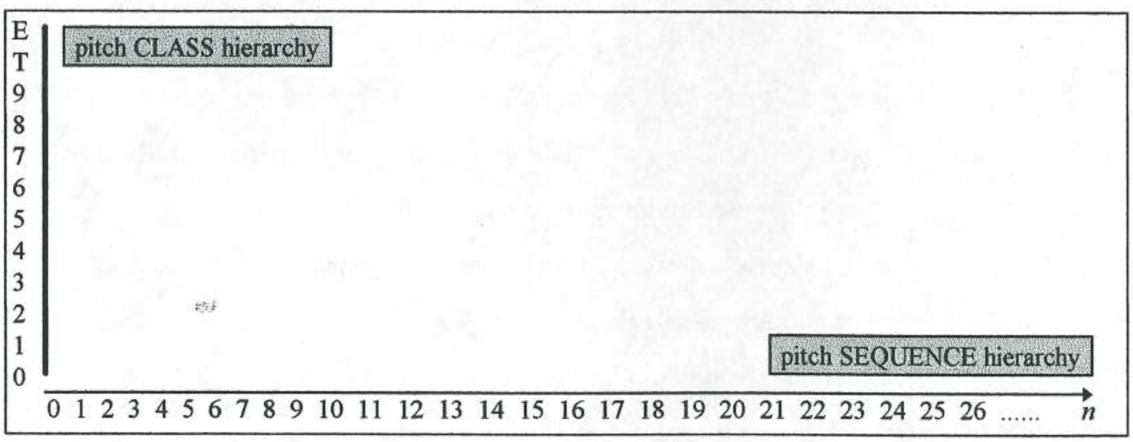
Evidence of this type of gestural melody can be found in the work of almost every Baroque, Classical and Romantic composer. There appears to be a potentially infinite number of such patterns possible.

**9. Sequence Filtering ("sequence layering", "spatial sequence operations")**

If we assume that we have an ordered pitch set (such as a scale) transfixed along a horizontal axis, we can assign an integer value to each pitch to define its **position,** both within the set and along the horizontal axis **[ex. 9-1]**. We can then manipulate these integer values to create *sequences*, by mapping a transposable pattern such as [4,5,4,3,4,2,3,4] onto the set to create a *melodic figure.* **[ex. 9-2]**. The subject or episodic material of any Bach fugue is generally an excellent example of this melodic device.

In order to eliminate potential confusion immediately from the outset, let me make it absolutely clear that **the integer system being applied here is *not* a mod-12 system**. To further clarify this point:

- Integer representation of **pitch *class*** takes place on the **vertical** axis of hierarchy, which is limited by the finite number of total pitch classes. When dealing with *spatial sequence operations*, integers *DO NOT* represent pitch classes, or transpositions of pitch class or *interval content* along the vertical axis.
- Integer representation of **pitch *sequence*** takes place on the **horizontal** axis of hierarchy, which is potentially unlimited. When dealing with *spatial sequence operations*, integers *DO* represent *fixed positions of pitches within an ordered set along a horizontal axis*, and transposition is being applied to *sequence patterns* along the horizontal axis.



If you attempted to perform these *spatial sequence operations* while mistakenly assuming that the integers are being used to represent the characteristics of a mod-12 system, you would achieve incorrect results. The horizontal axis *cannot* be represented by a finite integer system. It must be represented by an open-ended, potentially infinite integer system, due to the fact that an ordered set can contain an unlimited number of characters.

Once the output of a sequence operation has also been transfixed horizontally, the resulting material can be treated as another ordered set. We can re-assign the integer values to again define the position of each pitch along the horizontal axis within the new set **[ex. 9-3]**. It is then possible to apply yet another transposable sequence pattern such as [0,1,2,3,4,3,1,2,3,4] to this new source **[ex. 9-4]**. This is a process I term *"sequence filtering"* (or *"sequence layering"*), in the sense that one is filtering the results of the primary melodic figuration pattern through a secondary melodic figuration pattern, layering one on top of the other. To put it another way, **sequence filtering involves mapping a secondary sequential algorithm onto the ordered results of a previous sequential algorithm**. This technique works especially well with 12-tone rows; for illustration purposes, let us assume the pattern [1,2,3,0,4,5,2,4,6] **[ex. 9-5]**. It is worth noting that this process resembles certain rudimentary aspects of basic cryptography, as it can be applied to *any* ordered set of characters for the purpose of *scrambling* or *masking* the arrangement of the original set.

**10. Sequence Interpolation**

If several unique and different sequential figuration patterns are generated, whether from a single ordered set **[ex. 10-1]** or multiple ordered sets **[ex. 10-2]**, it is possible to isolate and extract segments from each pattern and interpolate them with one another **[ex. 10-3]**. This process generates a sort of "hyper-sequence", comprised not of a single mathematical pattern, but a combination of mathematical patterns. The resulting combined pattern can then be treated as a single large motivic "cell", and employed accordingly.

## 11. Compound 12-tone rows

A *compound 12-tone row* is an ordered pitch set which represents at least one iteration of every pitch-class, and contains a total of more than 12 pitches. The fundamental premise of compound row construction is that one chromatic aggregate need not be completed before another is introduced, as long as every required aggregate is resolved within the space occupied by the final ordered set. There are two types of compound rows:

- **symmetrical,** and
- **asymmetrical**.

***Symmetrical*** compound rows contain ($12x$) total pitches, representing $x$ iterations of every pitch class **[ex. 11-1]**. To put it another way, symmetrical compound rows are made up of $x$ chromatic aggregates, or $x$ combined 12-tone rows. A symmetrical compound row is not necessarily made up of $x$ 12-tone rows merely strung together end-to-end; the most organic way to construct one is to intermingle the contents of several rows by interpolation of individual pitches.

***Asymmetrical*** compound rows contain ($12x+n$) total pitches, where

$$[1 \leq n \leq 12x\text{-}1],$$

representing at least $x$ iterations of every pitch class, and *no more* than $2x$ iterations of any pitch-class **[ex. 11-2]**. Asymmetrical compound rows contain more iterations of some pitch-classes than others by adding $n$ extra pitches to $x$ chromatic aggregates, but never more iterations of any one pitch-class than *twice the original number of aggregates*. In other words, asymmetrical compound rows require the resolution of $x$ chromatic aggregates, while allowing the additional presence of $x$ partial or *unresolved* chromatic aggregates. This permits a limited degree of pitch-class favoritism, and allows greater freedom in constructing the melodic contour of the row, without completely abandoning the dodecaphonic aesthetic.

Both *symmetrical* and *asymmetrical* compound rows should avoid the appearance of overt centricity. Every iteration of any pitch-class should be separated from every other iteration of that same pitch-class by at least ($x$+1) places within the final ordered set.

**12. Compound rows by set interpolation**

Several 12-tone rows can be broken down into smaller sets, and intermingled with one another by interpolation of these component sets [**ex. 12-1**]. Again, this operation permits greater freedom in constructing melodic contour than conventional 12-tone technique. One chromatic aggregate need not be completed before another is introduced, as long as every required aggregate is resolved within the space occupied by the final ordered set.

**13. Row arpeggiation**

*Row arpeggiation* is a variation of the previous technique, the fundamental difference being that resolution of chromatic aggregates is not necessarily a consideration in this process. It is an organic and intuitive operation that can actually be practiced with any type of ordered pitch set, although it works best with 12-tone rows. The basic idea is similar to expanded functional harmonic progressions: simply treat every pitch in a chosen ordered set as a point of resolution, and apply new melodic material to it by interpolation [**ex. 13-1**]. This algorithm instigates the creation of melodic contours and pitch-set relationships that might not present themselves in any other context.

**14. Contour Cycling**

In this final melodic algorithm, a graphic model of some kind is devised as a visual guide for constructing melodic contours. Various pitch sets are then mapped onto this contour model an indeterminate number of times, until the desired amount of pitch material is generated [**ex. 14-1**].

A contour cycle produces the most satisfactory results if it is approached as a large-scale asymmetrical compound row. The eventual resolution of as many chromatic aggregates as possible is extremely desirable. The chromatic checklist found in **appendix 2** becomes indispensable in this operation.

## V. Algorithms for Manipulating Rhythm

### 15. Meter Rows

As with pitch, it is possible to govern cyclic patterns of rhythmic meter via ordered integer sets. These sets may contain as few or as many characters as one chooses, and may be used to represent meters of whatever note denomination (half, quarter, eighth, *etc.*) is desired. The basic rule of thumb I use in constructing a meter row is that **every integer should add up to a prime number with the integers in adjacent positions within the set**. The following is a good example of such a set:

[7,4,8,5,6,2,9,6,4,3,6,8,7,4,6,9,8] **[ex. 15-1]**

### 16. Additive and Subtractive Polyrhythms

Given any simple rhythmic pattern **[ex. 16-1]**, it is possible to generate polyrhythms by interpolating discrete rhythmic units into the existing pattern **[ex. 16-2]**, or removing iterations of discrete rhythmic units from the existing pattern **[ex. 16-3]**. **This is a traditional compositional practice; I include it here only as a reminder not to neglect rhythmic considerations while working with harmonic and melodic algorithms.** The greatest challenge to any composer is making these polyrhythmic patterns *idiomatic* for performance purposes, which can only be achieved by experimentation.

1/15/05 – BASIC EXPLANATION and DEMONSTRATION of SEQUENCE FILTERING

| Filter array | (horizontal axis) 0 1 2 3 4 5 6 7 8 9 T E — Row | PITCH CLASS OUTPUT |
|---|---|---|
| 0 2 3 5 7 | 4 3 6 7 T 9 E 2 0 8 1 5 | [4 6 7 9 2] |
| 1 3 4 6 8 | 4 3 6 7 T 9 E 2 0 8 1 5 | [3 7 T E 0] |
| 2 4 5 7 9 | 4 3 6 7 T 9 E 2 0 8 1 5 | [6 T 9 2 8] |
| 3 5 6 8 T | 4 3 6 7 T 9 E 2 0 8 1 5 | [7 9 E 0 1] |
| 4 6 7 9 E | 4 3 6 7 T 9 E 2 0 8 1 5 | [T E 2 8 5] |
| 5 7 8 T 0 | 4 3 6 7 T 9 E 2 0 8 1 5 | [9 2 0 1 4] |
| 6 8 9 E 1 | 4 3 6 7 T 9 E 2 0 8 1 5 | [E 0 8 5 3] |
| 7 9 T 0 2 | 4 3 6 7 T 9 E 2 0 8 1 5 | [2 8 1 4 6] |
| 8 T E 1 3 | 4 3 6 7 T 9 E 2 0 8 1 5 | [0 1 5 3 7] |
| 9 E 0 2 4 | 4 3 6 7 T 9 E 2 0 8 1 5 | [8 5 4 6 T] |
| T 0 1 3 5 | 4 3 6 7 T 9 E 2 0 8 1 5 | [1 4 3 7 9] |
| E 1 2 4 6 | 4 3 6 7 T 9 E 2 0 8 1 5 | [5 3 6 T E] |

↑ Row

- By sequencing the row along its horizontal axis, we actually interpolate pitch class material into the space between one position in the ordered set, and the next.
- As long as we sequence through the full 12 positions of the row, no matter *what* sequence we use, we will still end up with as many aggregates as integers in the sequence.

David M. Shaw II

# COMPOUND ROWS

David M. Sloore

ex. 11-1 (symmetrical) (X = 3)

Row 1

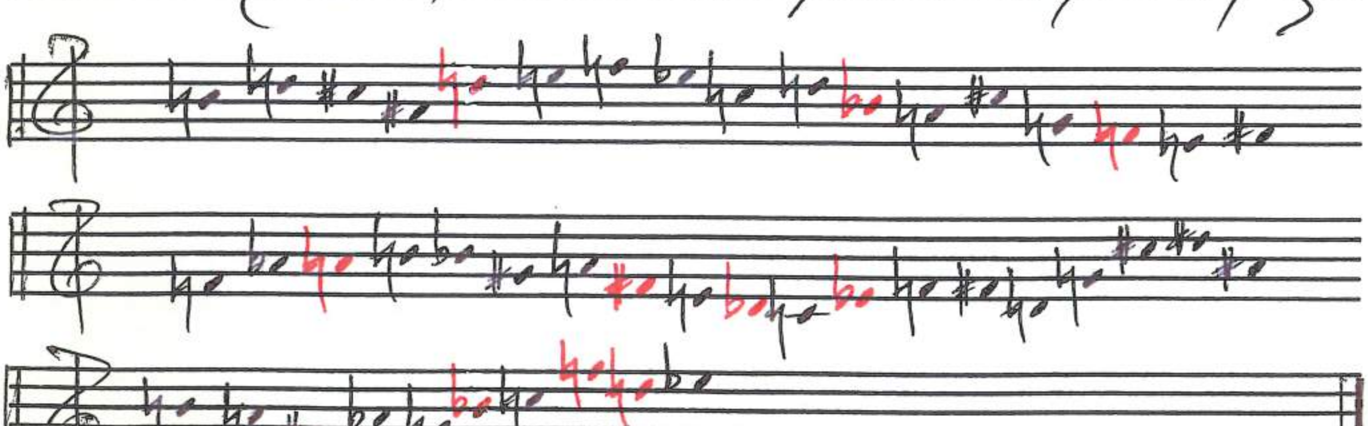
Row 2

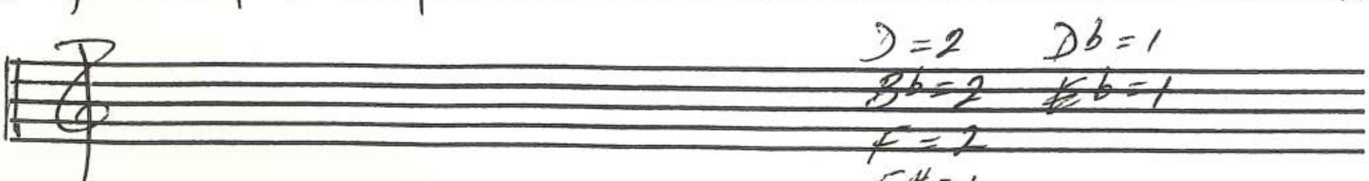
Row 3

COMPOUND (by interpolation)

ex. 11-2 (asymmetrical) (X=3, n=10)

{ adding pitches to previous results to produce asymmetry }





D = 2  Db = 1

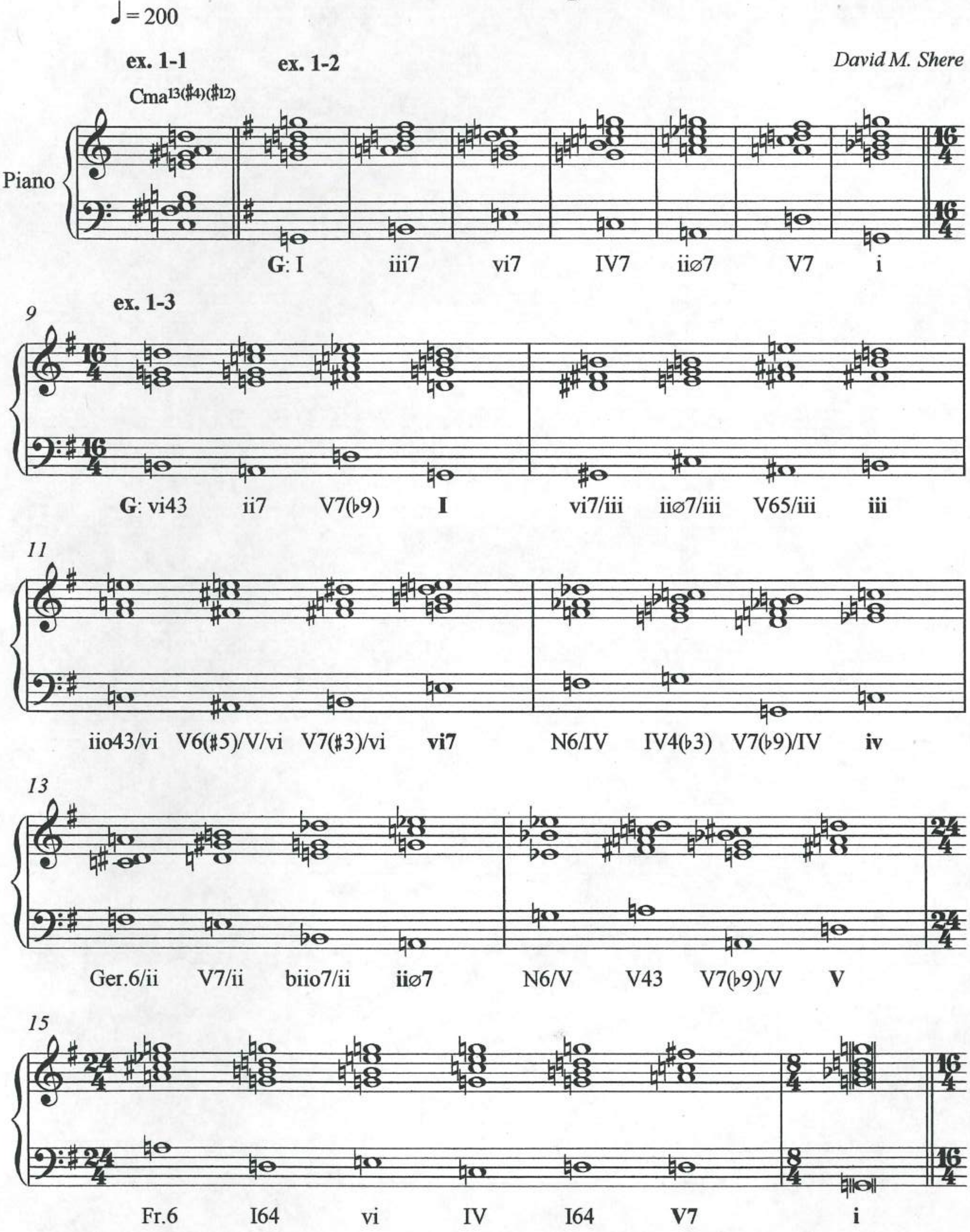
Bb = 2  Eb = 1

F = 2

F# = 1

A = 1

# *Appendix 1*
## *Musical Examples*



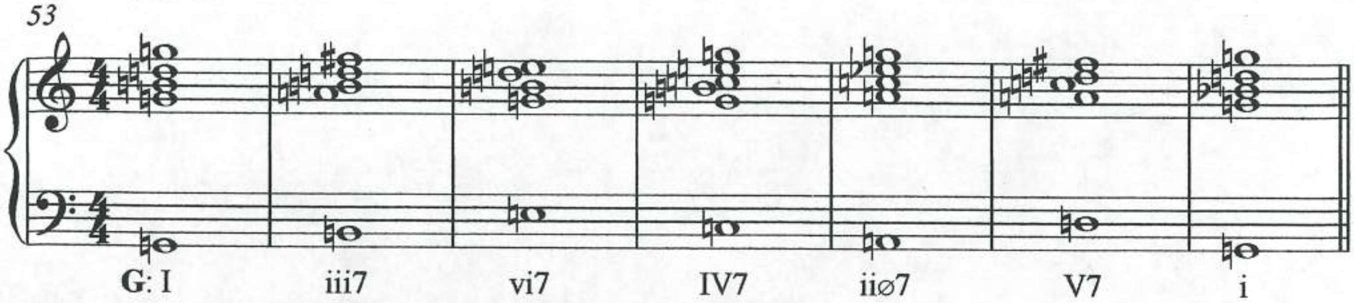
ex. 1-4

[certain passing chord functions become too ambiguous for useful analysis at this point]


29
N6(♭7)(♭♭4)/IV
IV4(♭3)(♭2)
31
V7(♭9)/IV
iv(♭9)
33
Ger.6/ii
V7(♭8)/ii
35
♭iio7/ii
iiø7
37
N6(♮7)/V
V43(♯7)
39
V7(♭9)/V
V(♭9)


41
Fr.6
I64(♯7)(♭5)
43
vi(♯7)
IV(9)(♭6)
45
I64(5)(♭2)
V7(♭5)
47
i
49
ex. 2-1
C: I76
v7/iii
♭III76
V7(♭9)/ii
v6(♯5)
Fr.6/♭II
vi6(♮5)/♭II
V6(♯4)32/vi
51
♭VI7
ii7(♭9)
iiø7/V/iii
♭iii42
(enharmonic spelling)
Fr.6

**ex. 3-1**




**ex. 3-2**



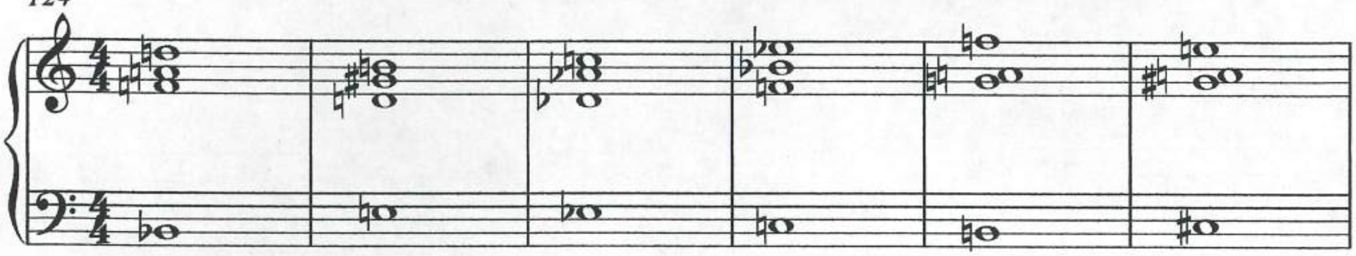
**ex. 3-3**

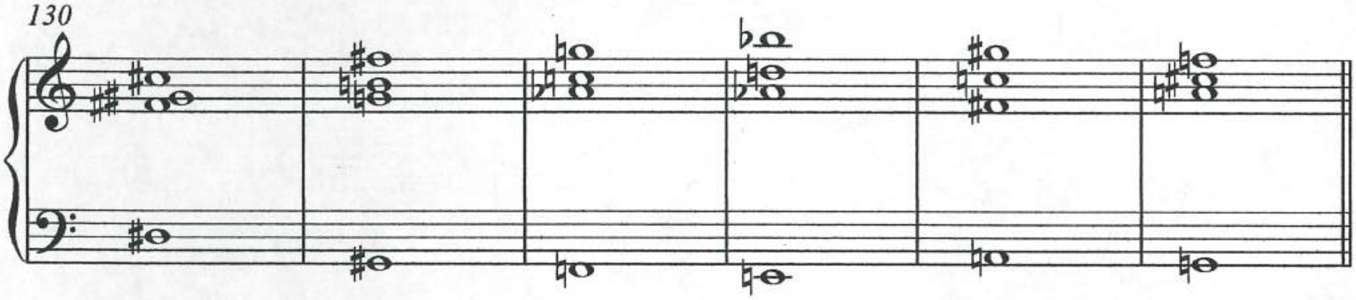

[the positions of these two chords have been switched for voice-leading purposes]


85
IV7
ii⌀7
92
[the positions of these two chords have been switched for voice-leading purposes]
V7
i
ex. 3-4
98
104
110
[original bassline altered to follow voice-leading]
116
[pitch content altered for voice leading]
48
4

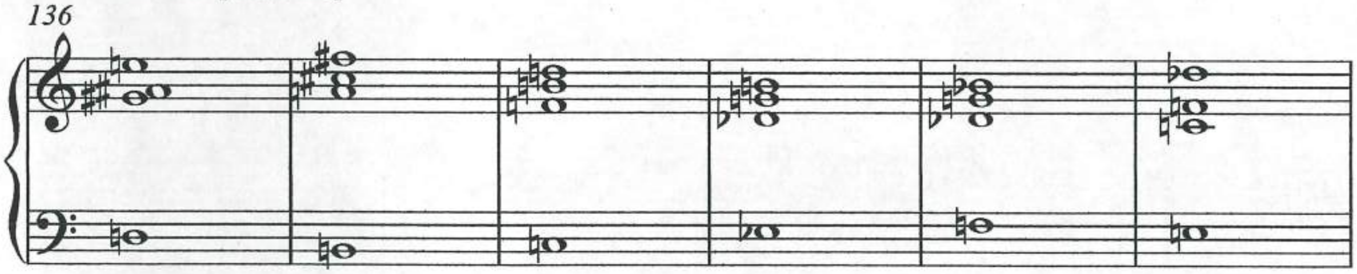
ex. 4-1

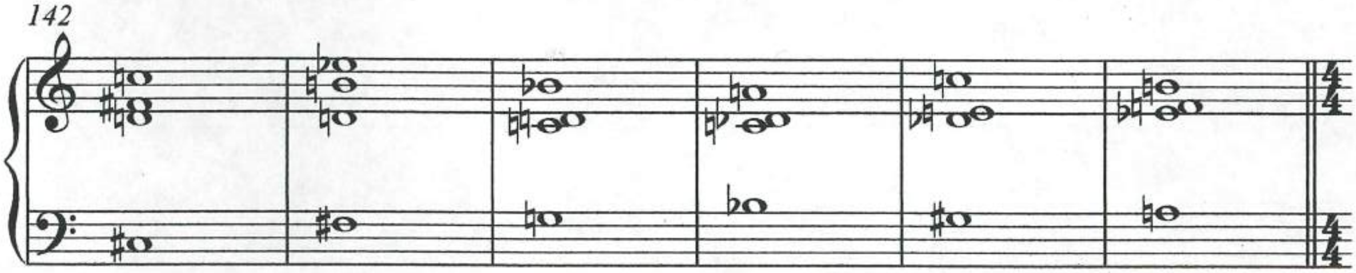

ex. 4-2 [soprano melody]



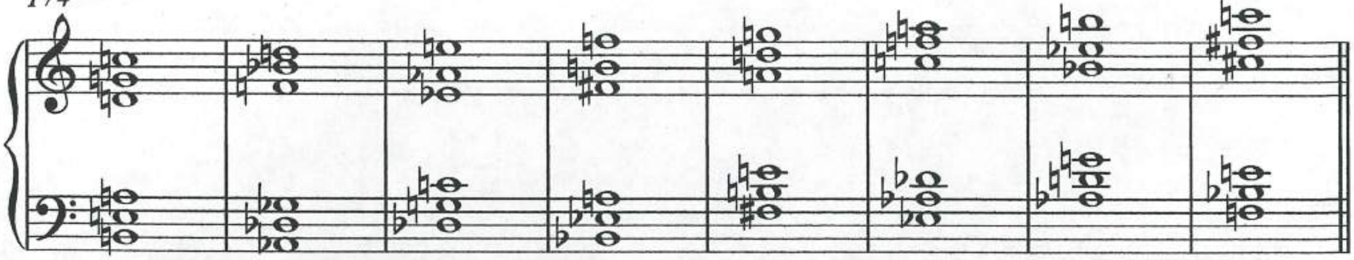

ex. 4-3 [bassline]



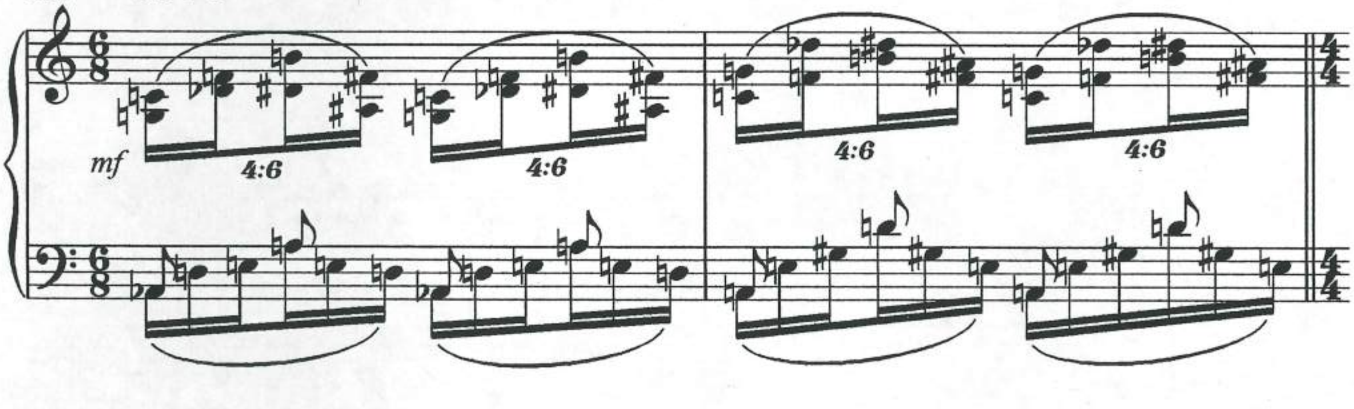

148
ex. 5-1
iiø7
♭VI9
V9/iii
iv(ma7)
♭VII7
V42/iii
IV7
Fr.6
V(ma7)
iiiadd9
153
iiø7
♭II7
Fr.6/V
iiø7
V7/vi
vi/♭II
V42/ii
iiø7/ii
viio/iii
Fr.6/ii
158
ex. 6-1
ex. 6-2
162
ex. 6-3
ex. 6-4
170
ex. 6-5

174 **ex. 6-6**



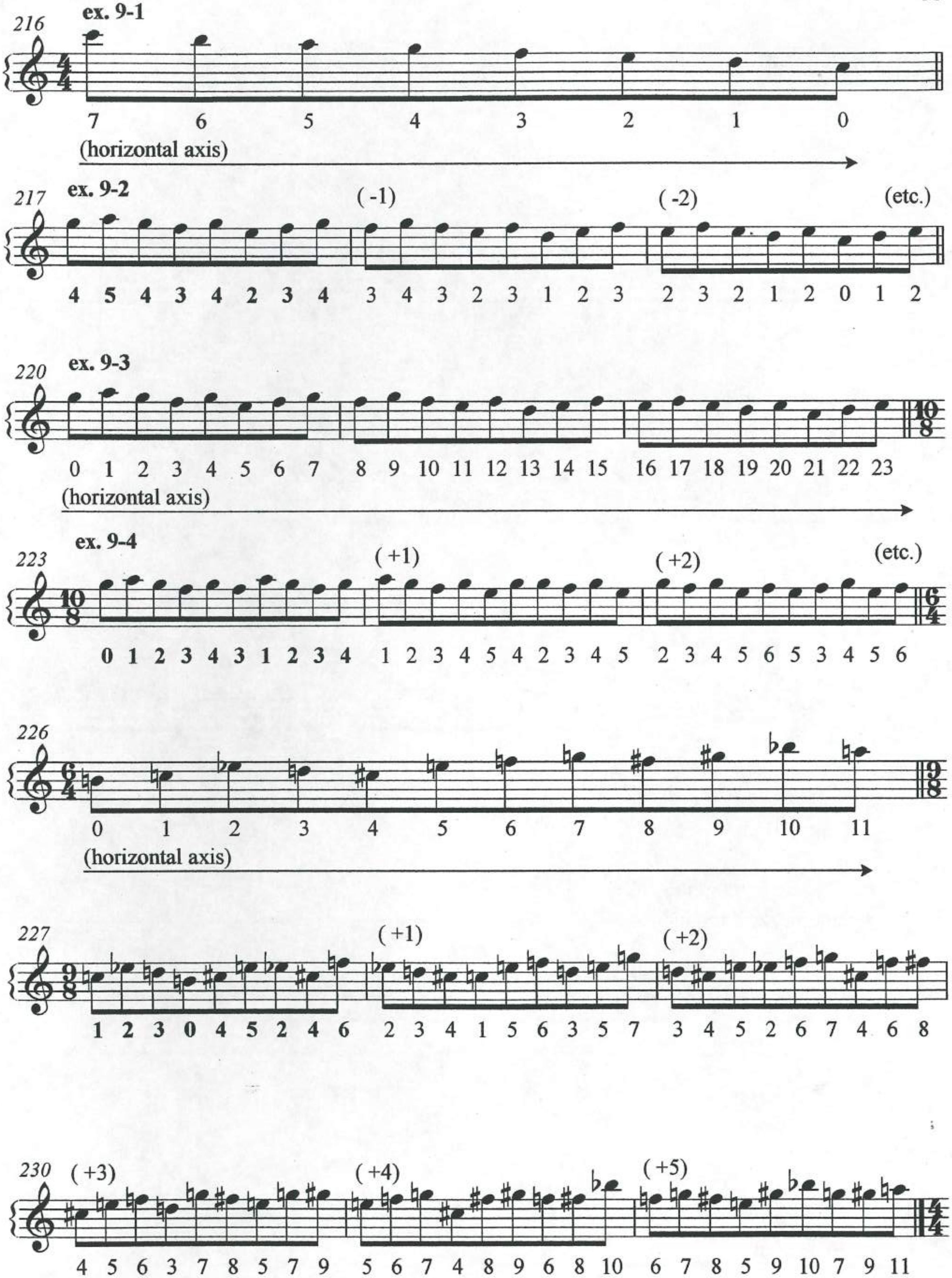
182 **ex. 6-7**

190 **ex. 7-1**




192 **ex. 7-2**


♩ = 350
ex. 8-1
195
198
201
ex. 8-2
204
ex. 8-3
206
ex. 8-4
208
210
212
214


216
ex. 9-1
7 6 5 4 3 2 1 0
(horizontal axis)
217
ex. 9-2
( -1)
( -2)
(etc.)
4 5 4 3 4 2 3 4 3 4 3 2 3 1 2 3 2 3 2 1 2 0 1 2
220
ex. 9-3
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
(horizontal axis)
223
ex. 9-4
( +1)
( +2)
(etc.)
0 1 2 3 4 3 1 2 3 4 1 2 3 4 5 4 2 3 4 5 2 3 4 5 6 5 3 4 5 6
226
0 1 2 3 4 5 6 7 8 9 10 11
(horizontal axis)
227
( +1)
( +2)
1 2 3 0 4 5 2 4 6 2 3 4 1 5 6 3 5 7 3 4 5 2 6 7 4 6 8
230
( +3)
( +4)
( +5)
4 5 6 3 7 8 5 7 9 5 6 7 4 8 9 6 8 10 6 7 8 5 9 10 7 9 11

**ex. 10-1**


**ex. 10-2**


**ex. 10-3**



**ex. 11-1** (see color-coded sketch for clarification)

symmetrical compound row


**ex. 11-2**

249 asymmetrical compound row

250

**ex. 12-1**

251 row 1 row 2

253 compound row by set interpolation

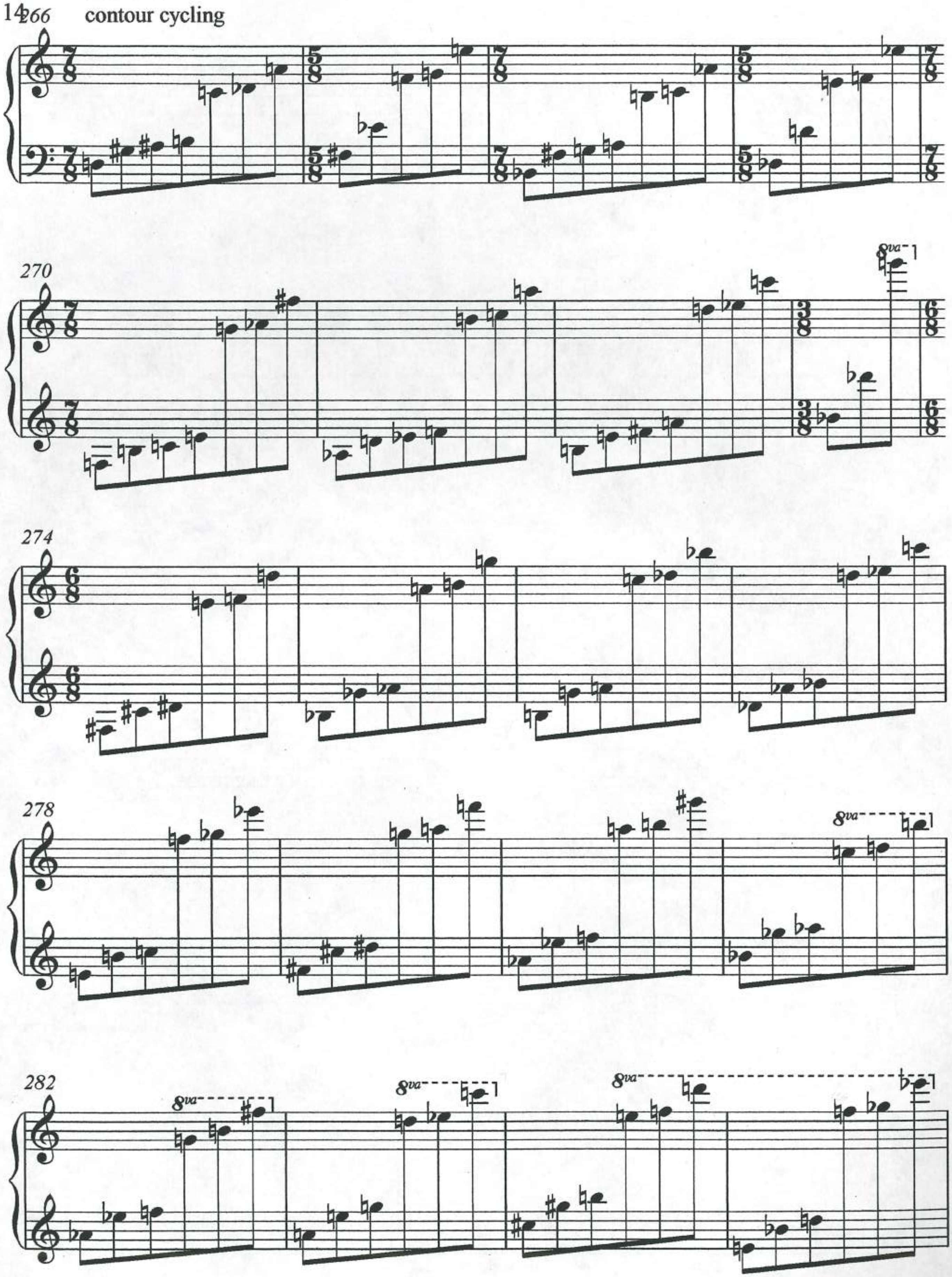
**ex. 13-1**

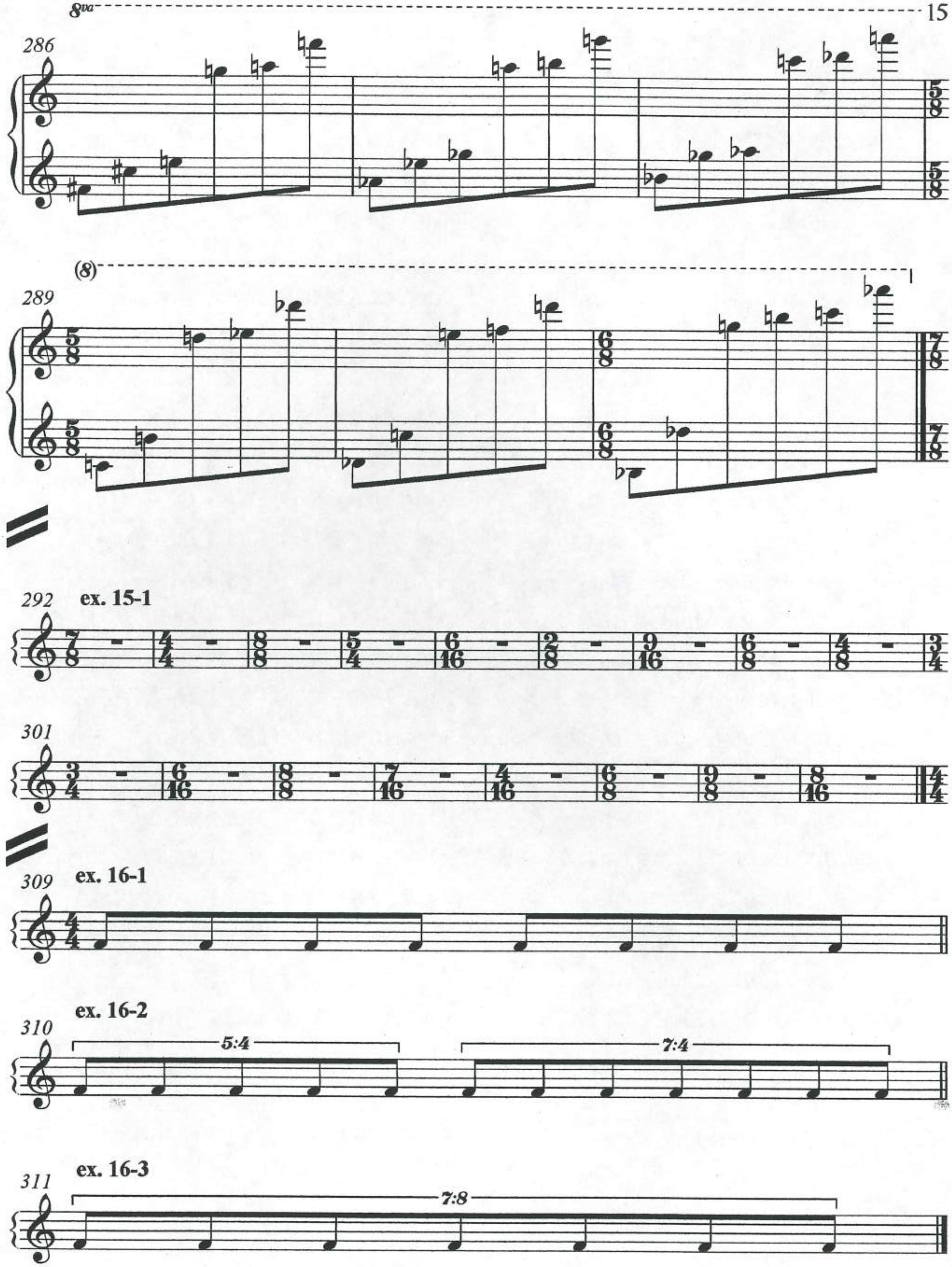
254 row arpeggiation

260

**ex. 14-1**

contour cycling




8va
286
(8)
289
292
ex. 15-1
301
309
ex. 16-1
310
ex. 16-2
5:4
7:4
311
ex. 16-3
7:8

## APPENDIX 2

***PITCH CHECKLIST TEMPLATE***

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

**Ab A Bb B C Db D Eb E F Gb G**

***INTEGER CHECKLIST TEMPLATE***

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

0 1 2 3 4 5 6 7 8 9 T E

## APPENDIX 3: BIBLIOGRAPHY (Recommended Reading)

Aldwell, Edward and Carl Schacter. **Harmony and Voice leading: 3rd Edition.** Wadsworth Group/Thomson Learning: Belmont CA, 2003.

Benjamin, Thomas, Michael Horvit and Robert Nelson. **Techniques and Materials of Tonal Music, With an Introduction to Twentieth-Century Techniques: Fourth Edition.** Wadsworth, Inc.: Belmont CA, 1992.

Jay, Stephen, Donald J. Rauscher and Justine Shir-Clif. **Chromatic Harmony.** The Free Press/MacMillan: New York NY, 1965.

Roads, Curtis. **The Computer Music Tutorial.** MIT Press: Cambridge MA, 1996.

Slonimsky, Nicholas. Thesaurus **of Scales and Melodic Patterns.** Schirmer Books/Amsco Publications: New York NY, 2003.

Straus, Joseph N. **Introduction to Post-Tonal Theory: Second Edition.** Prentice-Hall, Inc.: Upper Saddle River NJ, 2000.